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Effect of Coulomb interaction on GaAs quantum computer performance

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Abstract. Interaction between the quantum bits strongly limits quantum computer performance while using large quantum registers. We investigated influence of such interaction for recently proposed quantum computer based on GaAs quantum dots with built-in barrier. Our results show that such interaction could greatly reduce computational fidelity.

Introduction

There has been a tremendous progress in quantum computation theory since the pioneering work by R. Feynman [ ]. One of the most important reasons why the experimental realization of practically useful large scale quantum computer is not attained until now is attitude of quantum computers (compared to classical ones) to various types of errors. Hopefully there are known quantum error correction procedures, which help to correct errors which occurred simultaneously in single quantum bit (qubit) or in few qubits due to interaction with environment or imprecise implementation of local gates.

In 1998 J. Gea-Banacloche [ ] revealed the significance of another source of errors: internal interaction in quantum computer between neighbour qubits. The interaction serves to entangle qubits if necessary, so it should be switched on every time while implementing two-qubit gate and be switched off otherwise. But the accuracy of switcher amplitude could not be generally higher than several orders of magnitude. We should also control moments of switching off/on with such accuracy. Therefore there is weak unavoidable interaction between qubits any time. J. Gea-Banacloche [ ] noted that even such a weak interaction leads to errors which could completely destroy quantum computer performance in large scale quantum registers. These errors differ from common few-qubit errors since they originate from internal qubit-qubit interaction rather than from influence of noisy environment. They conserve coherence of quantum computer but spreads over whole quantum register and make initially unentangled blocks of qubits to entangle each other. In 1999 it was pointed out by the same author [ ] that common error correction methods do not solve the problem since these procedures imply that the probability of whole quantum register to be entangled due to errors during time of performing one of basic gates (we will denote this time as τ) is negligible.

1. Errors due to qubit-qubit interaction

As it was shown in [ ] the interaction between neighbour qubits i and j frequently leads to Hamiltonian which in the basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ will have a matrix like the following

$$H_{ij} = \begin{pmatrix} a^2 & 0 & 0 & 0 \\ 0 & ab & 0 & 0 \\ 0 & 0 & ab & 0 \\ 0 & 0 & 0 & b^2 \end{pmatrix}. \qquad (1)$$

The inequality $a \neq b$ results in nonadditive interaction energy. It is convenient to split interaction Hamiltonian into additive H_A and nonadditive parts H_N, where

$$H_A = \begin{pmatrix} a^2 & 0 & 0 & 0 \\ 0 & \frac{a^2+b^2}{2} & 0 & 0 \\ 0 & 0 & \frac{a^2+b^2}{2} & 0 \\ 0 & 0 & 0 & b^2 \end{pmatrix}, \tag{2}$$

$$H_N = \hbar\delta\omega \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{3}$$

where

$$\delta\omega = \frac{(a-b)^2}{2\hbar}. \tag{4}$$

The dimensionless parameter δ ($\delta = \tau\delta\omega$) can be used to evaluate entanglement during one computing step. The action of H_A does not entangle qubits. Moreover, by going to an interaction picture with state $|0\rangle$ having additional energy $a^2/2$ and state $|1\rangle$ having additional energy $b^2/2$ additive part can be effectively removed. The numerical value of factor $\delta\omega$ in H_N depends on qubits being used. In some cases non-additive part can contain also off-diagonal terms, whose influence on computation performance are similar to diagonal terms.

Fidelity F of quantum computer is

$$F = |\langle\Psi_{\text{ideal}}|\Psi_{\text{out}}\rangle|^2, \tag{5}$$

where $|\Psi_{\text{ideal}}\rangle$ is a result of ideal quantum computer, $|\Psi_{\text{out}}\rangle$ is a result of quantum computer being compared with the ideal one. For the calculation we use algorithm designed to add two n-bit numbers [9]. The computer consists of three registers, a, b, and c, of n, $n+1$, and n bits, respectively; and of $P = 2(4n-1)$ gates (CNOT and Toffoli gates). Computer action is the transfer $|a, b, c\rangle \rightarrow |a, a+b, c\rangle$, where $|c\rangle$ is zero ancilla register. The fidelity depends on the numbers being added. So we average the fidelity over b, using a in the fully entangled state:

$$F_{av} \equiv \frac{1}{2^n} \sum_{b=0}^{2^n-1} F(b). \tag{6}$$

Dependence of average fidelity versus scaled interaction strength $\sigma = \frac{1}{2}P\delta\sqrt{3n}$ is shown in Fig. 1.

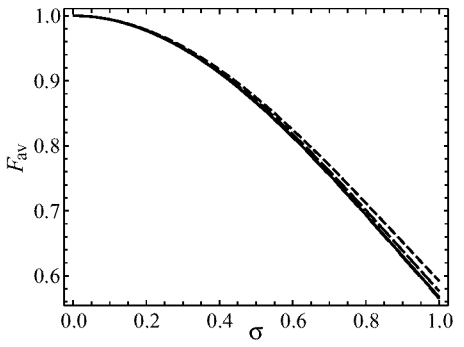

Fig. 1. Average fidelity versus scaled interaction strength. Case of dipole-dipole interaction. Solid curve, $n = 2$; short dashed curve, $n = 3$; dashed curve, $n = 4$; long dashed curve, $n = 5$.

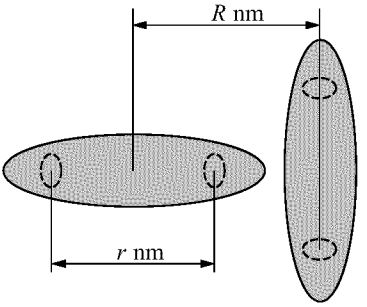

Fig. 2. GaAs quantum dot two-qubit structure.

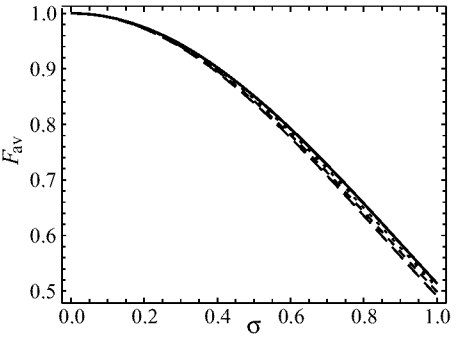

Fig. 3. Average fidelity versus scaled interaction strength. GaAs quantum dot quantum computer. Solid curve, $n = 2$; short dashed curve, $n = 3$; dashed curve, $n = 4$; long dashed curve, $n = 5$.

We investigated influence of Coulomb qubit-qubit interaction on performance of GaAs quantum dot quantum computer proposed in [ ]. The geometry of the structure is shown in Fig. 2.

Parameter δ of the structure for such interaction goes from depends on geometrical parameters and the material (GaAs) of the structure. For calculation we used following parameters: $r = 10-14$ nm (the distance between minima of the qubit-forming potential), $R = 20-50$ nm (the distance between qubits), $\kappa = 12.85$ (dielectric permittivity in GaAs). CNOT gate operation time $\tau = 1-100$ ns [ ].

Dependence of average fidelity versus scaled interaction strength with such interaction hamiltonian is shown in Fig. 3.

2. Conclusions

Operation of GaAs quantum computer performing adder algorithm was simulated in presence of Coulomb qubit-qubit interaction. Our simulation shows that in a large-scale quantum register ($n > 100$) qubit-qubit interaction could completely destroy quantum computer performance. Consequently use of multi-qubit error avoiding procedures in large-scale quantum computers with interacting qubits like firstly proposed algorithm [ ] is obligatory to maintain steady operation.

Acknowledgments

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